

# Transport blocking and topological phases using ac-magnetic fields

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We analyze electron dynamics and topological properties of open double quantum dots (DQDs) driven by circularly polarized ac-magnetic fields. In particular we focus on the system symmetries which can be tuned by the ac-magnetic field. Remarkably, we show that in the electron spin resonance (ESR) configuration, where the magnetic fields in each dot oscillate with a phase difference of  $\pi$ , charge localization occurs giving rise to transport blocking at arbitrary intensities of the ac field. The conditions for charge localization are obtained by means of Floquet theory and related with quasienergies degeneracy. We also demonstrate that a topological phase transition can be induced in the adiabatic regime for a phase difference of  $\pi$ , either by tuning the coupling between dots or by modifying the intensity of the driving magnetic field.

## INTRODUCTION

The study of topological features in driven systems is a fascinating topic, because of the emergence of non-trivial topologies once the system interacts with external driving fields[1–3]. Most of the topological features observed have been obtained under the assumption of adiabatic evolution, but also other interesting dynamical effects have been predicted in the non adiabatic regime [4–7]. Among them, electron spin locking induced by linearly polarized ac magnetic fields [5] or spin blockade induced by ac magnetic fields[8] have been recently proposed. Therefore, investigation of different regimes in driven systems, and the interplay between ac-fields, spatial symmetries and, topological features in quantum systems becomes a very promising field which could provide different mechanisms to manipulate electron charge and spin in confined systems. Spin qubits can be produced in confined systems as quantum dots, and their manipulation by driving the system with ac fields has been one of the most recently active topics, both experimentally[9–13] and theoretically[7, 8, 14–17].

In this work we analyze the electron dynamics and current through a double quantum dot (DQD) coupled to contacts and driven by circularly polarized magnetic fields. We show that the driving field is able to induce charge localization (CL) and then transport blocking for arbitrary field intensity at electron spin resonance (ESR) configuration. This localization effect depends critically on the symmetries of the system and its appearance can also occur in larger size systems such as arrays of quantum dots, linear ions traps[18], optical lattices[19], and more generally, in tunnel coupled systems with a pseudo-spin degree of freedom. We will show that in our system, at resonance condition, CL occurs, if the ac magnetic field oscillates in phase opposition in the different dots. We discuss as well how CL is also reached off resonance for dots with different Zeeman energies at a particular value of the ac frequency, i.e. at the mean value of the Zeeman energy splittings of the dots. Coupling the system to leads and applying a dc voltage gives rise to elec-

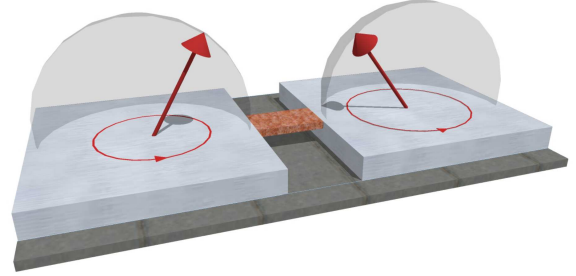


Figure 1: Schematic figure of a DQD coupled to circularly polarized magnetic fields oscillating in phase opposition

tronic current through the DQD and then to characterize CL from the current. We analyze the driven current by means of the Density Matrix (DM) formalism within the framework of the Floquet theory. It allows to effectively deal with the secular terms, and to characterize the effect of decoherence in the current.

In the last section we discuss how the topological properties of the system are modified in the case where the phase of the ac magnetic field within each dot differs in  $\pi$ . We demonstrate that a topological phase transition can be induced in the adiabatic regime by tuning the interdot coupling or by varying the intensity of the applied magnetic field.

## MODEL:

Our model describes a DQD in a strong Coulomb blockade regime, with just up to one extra electron, coupled to leads. The states spanning our DQD Hilbert space are:  $\{|\uparrow_L\rangle, |\downarrow_L\rangle, |\uparrow_R\rangle, |\downarrow_R\rangle, |0\rangle\}$ . The DQD is coupled to a magnetic field  $\mathbf{B}^i = (B_x^i(t), B_y^i(t), B_z^i)$  by magnetic dipole interaction  $g\mu_B \mathbf{S} \cdot \mathbf{B}(t)$  being  $i = L, R$ . The Zeeman splitting can be different for each dot due to different g-factors, differences in the nuclei polarization or inhomogeneities in the applied field. The ac-magnetic

field, that we assume circularly polarized, i.e.  $B_x^i(t) = B_{ac,x}^i \cos(\omega t + \phi_i)$  and  $B_y^i(t) = B_{ac,y}^i \sin(\omega t + \phi_i)$ , have a parameter  $\phi_i$  characterizing the phase difference between dots (Fig.1).

The Hamiltonian reads:

$$\begin{aligned} H(t) &= H_S(t) + H_B + V \\ H_S(t) &= H_0 + H_{ac}^B(t) + H_{dc}^B + H_{tLR} \end{aligned} \quad (1)$$

being  $H_0 = \sum_{i,\sigma} \epsilon_i d_{\sigma,i}^\dagger d_{\sigma,i}$  the dot Hamiltonian for the  $i$  dot,  $H_{ac}^B(t) = \sum_{i,\mu} B_{ac,\mu}^i(t, \phi_i) S_{\mu,i}$ , ( $\mu = x, y$ ) the coupling of the electronic spin with the external ac-field,  $H_{dc}^B = \sum_i B_z^i S_{z,i}$  the Zeeman splitting in the  $i$  dot, and  $H_{tLR} = t_{LR} \sum_{\sigma, i \neq j} d_{\sigma,j}^\dagger d_{\sigma,i}$  the interdot tunneling Hamiltonian ( $\mu_B = g = \hbar = 1$ ). The bath Hamiltonian  $H_B = \sum_{i,\sigma,K} \epsilon_{i,K} b_{i,K,\sigma}^\dagger b_{i,K,\sigma}$  represents fermionic reservoirs with operators  $b_{i,K,\sigma}^\dagger/b_{i,K,\sigma}$  for electrons with  $\epsilon_K$  energy and  $K$  momentum, and  $V = \sum_{i,\sigma,K} \Upsilon \left( b_{i,\sigma,K}^\dagger d_{i,\sigma} + h.c. \right)$  couples the DQD with the electron reservoirs. We consider Floquet theory because the dynamics can be easily extracted from the quasienergy spectrum, where degeneracies can be related with localization.

*Symmetries:* First we analyze the symmetries defined in the DQD, considering the parity symmetry (PS)  $\Pi : \{x \rightarrow -x\}$  and the generalized parity symmetry (GPS)  $\Pi_T : \{x \rightarrow -x, t \rightarrow t + T/2\}$ , which usually play an important role in driven systems, and classify the solutions according to a  $\mathbb{Z}_2$  group[5]. Applying the parity transformation to Eq.1, we obtain the condition for parity invariance:  $B_{z,L} = B_{z,R}$ ,  $\epsilon_L = \epsilon_R = 0$ ,  $\phi = 0$ , and  $B_{ac,\mu}^L = B_{ac,\mu}^R$ . Note that if we consider the generalized parity operation, an extra minus sign coming from the time dependent term shows up, leading to a non invariant Hamiltonian. The way to obtain  $\Pi_T$  invariance is by considering  $\phi = \phi_2 - \phi_1 = \pi$ , i.e. a  $\pi$  difference of phase between the ac-fields in each dot. This leads to a GP invariant Hamiltonian, but the PS is broken. Therefore the difference of phase  $\phi$  switches between  $\Pi$  and  $\Pi_T$  invariant systems.

In the present paper we shall consider  $\Pi$  and  $\Pi_T$  invariant Hamiltonians, just by tuning  $\phi$  to zero and  $\pi$  respectively. We also fix  $B_{ac,x}^L = B_{ac,y}^R$ , and just consider the possibility of asymmetric Zeeman splittings (breaking both, PS and GPS).

Also an internal symmetry in a single dot can be defined. In the linearly polarized case, a discrete  $\mathbb{Z}_2$  symmetry is present due to the time dependence of the field intensity  $|\vec{B}(t)|$ . By contrary, in the circularly polarized case, the intensity  $|\vec{B}|$  is constant and the system presents a continuous symmetry for rotations along the  $z$ -axis  $U_R(\theta) = e^{-i\theta S_z}$ . Furthermore, in this last case a time translation  $t \rightarrow t'$  is equivalent to a rotation along the  $z$  axis. This difference in the internal symmetry of the single dot in the presence of magnetic fields with dif-

ferent polarizations leads to the lack of dynamical spin locking[5] in the circularly polarized case.

*Master equation for the reduced density matrix:* When a quantum system is coupled to a dissipative bath such as a fermionic reservoir, exchange of energy and information occurs, leading to decoherence. In order to analyze how CL induced by circular ac-magnetic fields is affected by decoherence, and how this is reflected in the current, we develop a master equation within the framework of Floquet formalism, that allows to analyze the current through the system. The influence of phonons can be analyzed in a similar way, and an analogous effect in the current is expected.

We start by considering the Liouville equation for the total DM in the interaction picture. After some manipulations, the master equation in the steady state for the reduced DM elements  $\rho_{\alpha\beta}$  becomes (see details in [20, 21] and Appendix):

$$\begin{aligned} i\epsilon_{\alpha\beta} \rho_{\alpha\beta} &= \sum_{\mu\nu} (R_{\nu\beta,\mu\alpha}^1(0) + R_{\alpha\mu,\beta\nu}^2(0)) \rho_{\mu\nu} \\ &\quad - R_{\alpha\mu,\nu\mu}^1(0) \rho_{\nu\beta} - R_{\nu\beta,\nu\mu}^2(0) \rho_{\alpha\mu}, \end{aligned} \quad (2)$$

within the Markov approximation and Floquet basis, and being  $R_{\alpha\beta,\mu\nu}^{1,2}(0)$  the transition rates due to the coupling with the contacts, which couple the different Floquet states (see Eq.7 in Appendix). In order to evaluate the current we calculate the time derivative of the number of electrons  $Q_L = (-e)(N_L + N_{dot,L})$  in the left dot  $N_{dot,L} = \sum_{\sigma} d_{\sigma,L}^\dagger d_{\sigma,L}$ , plus the left reservoir  $N_L = \sum_{K,\sigma} b_{K,L,\sigma}^\dagger b_{K,L,\sigma}$ . Hence the average current is:  $\langle I \rangle = \langle \dot{Q}_L \rangle = i\langle [H(t), Q_L] \rangle$ .

## RESULTS:

We calculate the occupation probabilities in different states and the current for different field configurations. The results are compared with those of the closed system. We also consider different temperatures and analyze its effect on the decoherence.

There are two cases to be considered, both in resonance condition  $B_{z,L} = B_{z,R} = \omega$ . First, we deal with a  $\Pi$  invariant Hamiltonian ( $\phi = 0$ ). In this case, the quasienergies are not degenerate for all  $B_{ac}$  (see Fig.2, left), but just isolated crossings appear for certain values of the field intensity. The time evolution of the occupation probabilities in the closed system (not shown in the paper) indicates that CL does not occur for this field configuration independently of the field intensity and frequency, and the electrons oscillate back and forth between the dots. Our numerical calculation shows that all Floquet states become equally occupied in the steady state. At a finite voltage  $\mu_L - \mu_R \gg T, B_z, B_{ac}$ , the current reaches a finite constant value (Fig.3), which confirms the lack of CL.

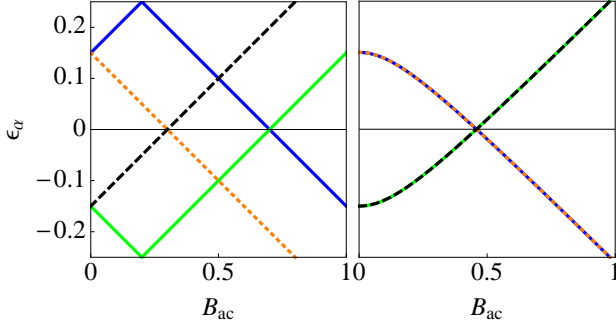


Figure 2: (Color online) Quasienergies vs  $B_{ac}$  for  $\phi = 0$  (left) and  $\phi = \pi$  (right), in resonance condition. The absence (presence) of  $\Pi_T$  symmetry for  $\phi = 0$  ( $\phi = \pi$ ), leads to non-degenerate (doubly degenerate) quasienergies. The degeneracy, present for all  $B_{ac}$  in the right figure, drives the system to CL.  $t_{LR} = 1/10$ , and  $B_{z,L} = B_{z,R} = \omega = 1/2$ .

If we consider a  $\Pi_T$  symmetry invariant Hamiltonian ( $\phi = \pi$ ), the quasienergies become doubly degenerated for all  $B_{ac}$  (Fig.2, right). The occupation probabilities show that the system is in CL regime (Fig.4).

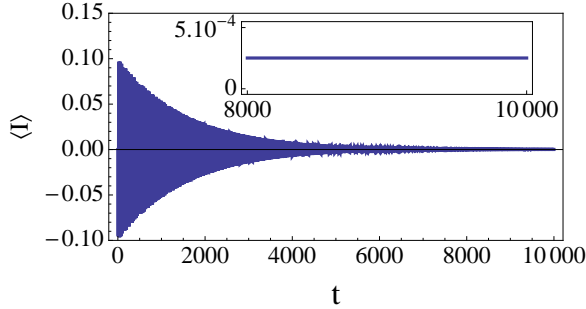


Figure 3: Current vs  $t$  for the case of  $\Pi$  symmetry ( $\phi = 0$ ) at resonance condition.  $B_{z,L} = B_{z,R} = \omega = 0.5$ ,  $B_{ac} = 0.5$ ,  $t_{LR} = 0.1$ ,  $\Upsilon = 0.01$ ,  $T = 10^{-3}$  and  $\mu_L - \mu_R \gg T, B_z, B_{ac}$ . The inset shows the current vs time in the steady state  $\langle I \rangle \simeq 2.5 \times 10^{-4}$  ( $3.5 \times 10^{-2}$  pA). All values are in energy units of 100mT (5.788  $\mu$ eV). ( $\omega \sim 4.4$ GHz), the time unit scale is  $\sim 0.23$ ns.

The existence of  $\Pi_T$  symmetry allows the  $\mathbb{Z}_2$  classification of the quasienergies, and then the double degeneracy for all  $B_{ac}$ , driving the system to CL regime. The crossings obtained between quasienergies with the same  $\Pi$  or  $\Pi_T$  symmetry are allowed by the Wigner-Von Neumann theorem because of the continuous symmetry of a single dot in a circularly polarized magnetic field, but they do not affect the occupation probabilities of the Floquet states. The symmetry difference between this case, and the case of a linearly polarized magnetic field (i.e. where the single dot symmetry is discrete), is the reason for the absence of dynamical spin locking in the former.

In the present case, i.e., for circularly driven fields we found a very interesting result: CL is obtained for all values of  $B_{ac}$  intensity. The reason is that the quasienergies

manifold is degenerated for all  $B_{ac}$ . Localization is improved by increasing the field amplitude, as we will show below.

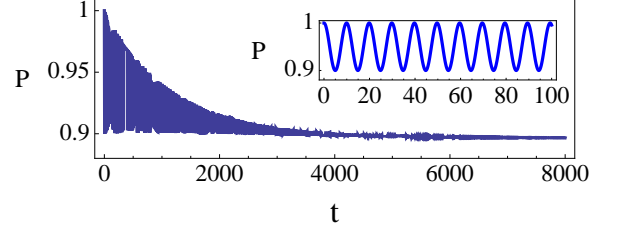


Figure 4: Occupation probability in the left dot  $P$  vs time for  $B_{ac} = 0.6$ , in presence of decoherence due to the leads. Note the spatial CL induced as a consequence of the quasienergy degeneracy between Floquet states with opposite  $\Pi_T$  symmetry. The inset shows the time evolution (Eq.3) for the closed system.  $B_{z,L} = B_{z,R} = \omega = 1/2$ ,  $\Upsilon = 0.01$ ,  $T = 10^{-3}$ ,  $t_{LR} = 1/10$  and  $\phi = \pi$ , all in units of 100mT (i.e. 5.788  $\mu$ eV) for  $\mu_L - \mu_R \gg 0$ .

It can be seen by direct comparison of Fig.4 and the inset, that the effect of decoherence due to the leads, is to remove the coherent oscillations without breaking the localization induced by the ac-field. In order to get a better insight of the effect of decoherence on CL we obtain analytically the localization probability for the closed system ( $\phi = \pi$  and  $B_{z,L} = B_{z,R} = \omega$ ):

$$P(t) = |\langle \uparrow_L | \Psi(t) \rangle|^2 + |\langle \downarrow_L | \Psi(t) \rangle|^2 \\ = 1 + \frac{2t_{LR}^2 \left( \cos \left( t \sqrt{B_{ac}^2 + 4t_{LR}^2} \right) - 1 \right)}{B_{ac}^2 + 4t_{LR}^2}, \quad (3)$$

Eq.3 shows that CL depends on the ratio  $B_{ac}/t_{LR}$ . From Fig.4 we assume that, in presence of decoherence the localization is given by the minimum value of the coherent oscillations in the closed system. We then can calculate the expected CL for the open system (Fig.5), which is given by:  $P_{\min} = \min(P(t)) = \Lambda^2 / (1 + \Lambda^2)$ , where  $\Lambda = B_{ac} / (2t_{LR})$ .

From the last results, we conclude that the existence of  $\Pi_T$  symmetry drives the system to charge localization, as  $\omega$  is in resonance with the Zeeman splitting. Once it is tuned off resonance, the quasienergies split, and localization is destroyed.

Now we consider asymmetric Zeeman splittings. This is a very common situation in real experiments, where Overhauser fields or different g-factors make difficult to achieve a symmetric configuration. If we calculate the Floquet spectrum for the Hamiltonian  $H_S(t)$  (1), and look for the necessary condition for degeneracy between quasienergies, we obtain:  $\omega = \omega_{Av} := (B_{z,L} + B_{z,R})/2$ . Note that now  $\Pi_T$  is always broken because of the asymmetric Zeeman splittings. For the frequency condition  $\omega = \omega_{Av}$ , the quasienergies overlap as in Fig.2 (re-scaled to the average Zeeman splitting), leading to CL regime at  $\phi = \pi$ .

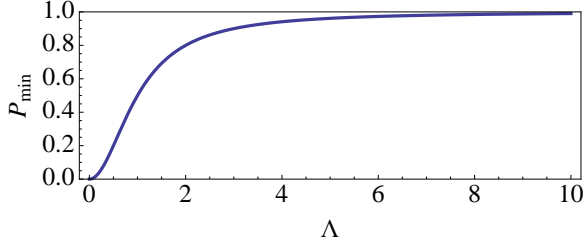


Figure 5:  $P_{\min}$  vs  $\Lambda$  in presence of coupling with the leads for the steady state ( $\phi = \pi$ ).

Finally Fig.6 shows  $\langle I \rangle / \omega$  for  $\phi = \{0, \pi\}$  and asymmetric Zeeman splittings.

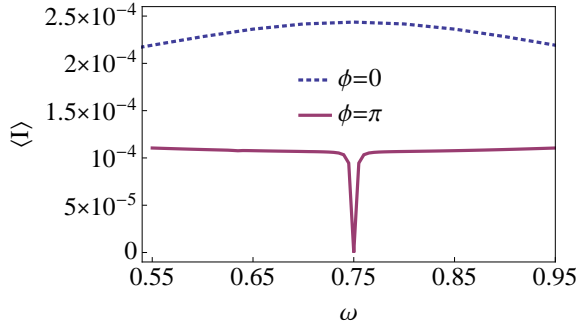


Figure 6: Current vs  $\omega$  in the steady state for asymmetric Zeeman splittings.  $\langle I \rangle$  drops to zero at  $\omega = \omega_{Av}$  ( $\omega_{Av} = 0.75$ ) for  $\phi = \pi$ , while for  $\phi = 0$  the current is finite.  $B_{z,L} = 0.5$ ,  $B_{z,R} = 1$ ,  $B_{ac} = 0.55$ ,  $t_{LR} = 0.1$ , and  $\Upsilon = 0.01$ . Different temperatures have been considered, but their effect just affect to the transitory regime.

This result demonstrates the critical difference in the current behavior for  $\phi = \{0, \pi\}$  at  $\omega_{Av}$ , where in the last case, CL is induced and the current drops to zero. Now, we will consider the adiabatic regime and we show that also the phase difference is critical and determines the topology of the system.

#### *Topological phases in the adiabatic limit:*

In the adiabatic limit, spin systems coupled to magnetic fields show a special feature. A non-trivial topological phase (characterized by integer numbers  $\mathbb{Z}$ ) arises for a single localized spin whenever the adiabaticity parameter  $\nu = \omega/2|\vec{B}|$  fulfills  $\nu < 1$ , being  $|\vec{B}| = \sqrt{B_z^2 + B_{ac}^2}$  [22]. As the  $\omega$  increases, the system is driven to the non-adiabatic regime, inducing a topological phase transition to a region where the topology is trivial.

In the present setup (for simplicity we consider  $B_{z,L} = B_{z,R}$ ), where the spin can oscillate between the two QDs, we characterize the new topological features due to the tunnel coupling. Considering both cases  $\phi = \{0, \pi\}$ , we

calculate the first Chern number  $c_1$  that fully characterizes the topology of the system, and the adiabaticity parameter  $\nu^\phi$  (see appendix for details). The result shows, in the  $\phi = 0$  case, that the tunneling does not change the topological properties in the adiabatic limit (same result as for a localized spin) with  $c_1 = \pm 1$  for all values of  $t_{LR}$ . In contrast, for the  $\phi = \pi$  case, it is possible to induce a topological phase transition within the adiabatic limit just by tuning  $\lambda = t_{LR}/|\vec{B}|$ . This can be observed in the first Chern number for each state:

$$c_1(\lambda) = \frac{1}{2} (1 + \text{sign}(1 - 2\lambda)) (1, -1, 1, -1) \\ = \begin{cases} (1, -1, 1, -1) & \forall \lambda < 1/2 \\ 0 & \forall \lambda > 1/2 \end{cases}$$

This result demonstrates that  $t_{LR} = |\vec{B}|/2$  is a critical point for the phase transition, and that within the adiabatic regime we can induce a topological phase transition just by tuning the intensity of the field.

In order to be in the adiabatic regime for the present configuration, it is required to be out of resonance  $\omega \ll B_z$ . Therefore the previous analysis concerning topology for  $\phi = \{0, \pi\}$  applies for regimes where CL cannot occur. It shows, as we commented previously, that different driving regimes in dynamical systems provide different ways to manipulate electrons.

## CONCLUSIONS:

We analyze the electron charge dynamics and the tunneling current through a DQD attached to contacts and driven by a circularly polarized magnetic field. We demonstrate both numerically and based in symmetry arguments that tuning the phase difference between the ac magnetic fields applied to the dots it is possible to achieve charge localization. This effect is robust and perdure in presence of decoherence, induced by the coupling with contacts. The effect of decoherence due to spin baths will be analyzed in a further work [23]. Finally, we demonstrate that out of resonance, in the adiabatic limit, we can drive our system to different topological phases by tuning the ratio  $t_{LR}/|\vec{B}|$ .

In summary, two level systems such as QDs coupled by tunneling and interacting with ac-magnetic fields show interesting features which depend drastically on the field polarization. The nature of the magnetic dipolar coupling  $-\vec{\mu} \cdot \vec{B}$  and the interplay between spatial and spin degrees of freedom coupled through the ac-magnetic field allow to achieve charge localization for arbitrary intensity of the ac magnetic field in the ESR regime. Also, in the adiabatic limit, we show that a topological phase transition can be induced. The present results demonstrate the huge horizon of possibilities obtained by combining ac magnetic fields and QD arrays, with direct applica-

tion in quantum computation with spin qubits and also in topological quantum computation, although the latter should be studied further for the case of non-abelian gauge theories.

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### Master equation. Detailed calculation

Following previous works [20, 21], we consider the Liouville equation for the density matrix and weak coupling with the leads. We calculate the reduced density matrix by tracing out the bath degrees of freedom. The assumptions at this point are the approximate factorization of the density matrix between bath and system, and Markov approximation. The density matrix of the bath is given by a fermionic thermodynamical ensemble in equilibrium.

We assume that our interaction Hamiltonian is linear in the system and lead operators:

$$V \equiv \sum_j F_j Q_j = \Upsilon \sum_{K,i,\sigma} \left( b_{K,i,\sigma}^\dagger d_{i,\sigma} - b_{i,\sigma,K} d_{i,\sigma}^\dagger \right) \quad (4)$$

being  $\Upsilon$  the coupling to the leads,  $F_{j,K}$  a lead operator, and  $Q_j$  a system operator ( $K$  is the momentum of the electron in the lead). We have considered multindex notation, being  $\pm j = (K, i, \sigma, \pm \xi)$ ,  $i = L, R$ ,  $\sigma = \uparrow, \downarrow$ , and  $\xi = \pm$  (this means annihilation/creation operator for the system). Identifying the coefficients we obtain  $Q_{i,\sigma,+} \equiv d_{i,\sigma}$ ,  $Q_{i,\sigma,-} \equiv -d_{i,\sigma}^\dagger$ ,  $F_{K,i,\sigma,+} \equiv b_{K,i,\sigma}^\dagger$ , and  $F_{K,i,\sigma,-} \equiv b_{K,i,\sigma}$ .

When we trace out the bath degrees of freedom, the terms involving the statistical ensembles can be written as:

$$G_1(t - \tau, t) = \sum_{j,l} \text{Tr}_B \left\{ F_j \tilde{F}_l(t - \tau, t) \rho_B \right\} \quad (5)$$

$$G_2(t - \tau, t) = \sum_{j,l} \text{Tr}_B \left\{ \tilde{F}_l(t - \tau, t) F_j \rho_B \right\},$$

The calculation of these terms, also integrating in  $\tau$ , leads to:

$$\begin{aligned} g_1(m\omega - \varepsilon_{\mu\nu})_{+,i} &= \pi D_i (1 - n_F(m\omega - \varepsilon_{\mu\nu} - \mu_i)) \\ &= g_2(m\omega - \varepsilon_{\mu\nu})_{-,i} \\ g_2(m\omega - \varepsilon_{\mu\nu})_{+,i} &= \pi D_i n_F(m\omega - \varepsilon_{\mu\nu} - \mu_i) \\ &= g_1(m\omega - \varepsilon_{\mu\nu})_{-,i} \end{aligned}$$

being  $D_i$  the density of states in the  $i$  lead, that we assumed to be constant in our regime,  $\mu_i$  the chemical potential, and  $n_F(E) = (1 + e^{E/(k_B T)})^{-1}$  the usual Fermi distribution function.

The rates of the master equation require the calculation of the matrix elements  $\langle \phi_\alpha(t) | Q_j \tilde{Q}_l(t - \tau, t) \rho_S(t) | \phi_\beta(t) \rangle$ , being  $|\phi_\alpha(t)\rangle$  the Floquet states of the closed system. Using Fourier series due to the periodicity of Floquet states, and defining  $Q(n)_{j,\alpha\beta} \equiv \frac{1}{T} \int_0^T dt e^{in\omega t} \langle \phi_\alpha(t) | Q_j | \phi_\beta(t) \rangle$ . We obtain the full master equation in Floquet basis:

$$\begin{aligned} (\partial_t + i\varepsilon_{\alpha\beta}) \rho(t)_{\alpha\beta} &= \sum_{\mu\nu} (R_{\nu\beta,\mu\alpha}^1(t) + R_{\alpha\mu,\beta\nu}^2(t)) \rho(t)_{\mu\nu} \quad (6) \\ &\quad - R_{\alpha\mu,\nu\mu}^1(t) \rho(t)_{\nu\beta} - R_{\nu\beta,\nu\mu}^2(t) \rho(t)_{\alpha\mu}, \end{aligned}$$

where  $R_{\alpha\beta,\mu\nu}^{1,2}(t) = \sum_M R_{\alpha\beta,\mu\nu}^{1,2}(M) e^{-i\omega M t}$ , and the Fourier coefficients are:

$$\begin{aligned} R_{\alpha\beta,\mu\nu}^1(M) &= |\Upsilon|^2 \sum_{i,\sigma,\xi} \sum_m Q(M+m)_{i\sigma\xi,\alpha\beta} Q(m)_{i\sigma\xi,\mu\nu}^* \\ &\quad \times g_1(\varepsilon_{\mu\nu} - m\omega)_{\xi,i} \\ R_{\alpha\beta,\mu\nu}^2(M) &= |\Upsilon|^2 \sum_{i,\sigma,\xi} \sum_m Q(M+m)_{i\sigma\xi,\alpha\beta} Q(m)_{i\sigma\xi,\mu\nu}^* \\ &\quad \times g_2(\varepsilon_{\mu\nu} - m\omega)_{\xi,i} \end{aligned} \quad (7)$$

The approximation considered in the paper  $R_{\alpha\beta,\mu\nu}^{1,2}(M) = R_{\alpha\beta,\mu\nu}^{1,2}(0)$ , can be assumed if the quasienergies are far from the boundaries of the Brillouin zone.

### Adiabatic limit

We define the adiabatic parameter following [24] as:

$$\begin{aligned} \omega_c &:= \sup \{ |A_{mn}(t)| : n \neq m = 1, \dots, 4 \}, \\ &\quad t \in [\tau_1, \tau_2] \subseteq [0, 2\pi/\omega] \\ A_{mn} &= \langle m; t | \frac{d}{dt} | n; t \rangle \\ \nu &:= \frac{\omega_c}{\omega_0} \end{aligned}$$

being  $\omega_0$  the level spacing to the first excited level and  $\tau$  the total time needed for a cycle. In the case  $\phi = 0 \rightarrow \omega_c^{\phi=0} = \Omega/2$  and the adiabaticity parameter is  $\nu^{\phi=0} = \Omega/2t_{LR}$  (taking  $\omega_0^{\phi=0} = t_{LR}$ ). The case  $\phi = \pi$  is very different because the level splitting to the first excited state is no longer given by the tunneling parameter. Hence,

$$\begin{aligned} \omega_0^{\phi=\pi} &= \frac{1}{2} \sqrt{|\vec{B}|^2 + 4t_{LR}^2 + 4t_{LR}B_z} \\ &\quad - \frac{1}{2} \sqrt{|\vec{B}|^2 + 4t_{LR}^2 - 4t_{LR}B_z}, \end{aligned}$$

$$\omega_c^{\phi=\pi} = \omega |\vec{B}| / \left( 2 \sqrt{|\vec{B}|^2 + 4t_{LR}^2} \right),$$

and

$$\nu^{\phi=\pi} = \frac{\omega_c^{\phi=\pi}}{\omega_0^{\phi=\pi}}$$

The adiabatic regime requires  $\nu^{\phi} \ll 1$ , and then small frequencies compared with all the other energy scales. For the adiabaticity condition we have assumed  $t_{LR} < B_z$ .

In conclusion we state that for frequencies small enough (far from resonance), varying the tunneling parameter, or the intensity of the magnetic field, a topological phase transition can be induced.

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